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E. J. Stone, Esq., Vice-President, in the Chair.

Boyd Moss, Esq., Clatford, Middlesex,

was balloted for and duly elected a Fellow of the Society.

On the Solar Parallax, derived from Observations of Mars at Ascension, in 1877. By David Gill, H.M. Astronomer at the Cape of Good Hope.

(*Abstract.*)*

Aided by a grant of 250*l.* from the Society, a like sum from the Government Grant Fund of the Royal Society, and with the loan of Lord Lindsay's Heliometer, Mr. Gill undertook an expedition to the Island of Ascension for the purpose of determining the parallax of the Sun, from observations of the diurnal parallax of *Mars*, during the favourable opposition of that planet in the year 1877.

After a brief history of the expedition, Mr. Gill describes the methods and instruments used, and gives the results of the operations by which the latitudes and longitudes of the observing stations were determined.

The stars used for comparison with *Mars* were observed in the meridian at thirteen of the principal Observatories. The result of a discussion of all these observations is given, and the proper motions are deduced from a discussion of all the old observations. A series of very accurate *preliminary* places is thus formed for comparison with the results of the heliometric

* The Memoir is being printed, and will appear in the next volume of the *Memoirs.*—Ed.

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triangulation. In the latter operation a night's work was so arranged that first a *standard distance* was measured, next three other distances in the triangulation, and then again the *standard distance*.

This *standard distance* was taken to be the angle between the stars $e-g$ for 1877·0. An approximate value of this angle (7054″·80) was adopted, the true value being assumed

$$7054''.80 + \Delta S.$$

From each measure of this *standard distance* the scale value was deduced in terms of ΔS, and from the different values so found the scale value was interpolated for the epoch of each observation. Thus in all the observations of the triangulation the deduced distances were affected systematically only by

$$\Delta S \times \frac{\text{measured distance}}{\text{standard distance}}$$

and by such errors as might exist in the heliometer by which the scale value would be different for the measurement of large and small angles. For example, if the radius of the cylindrical slides in which the segments move is different from the focal length of the object-glass, an error depending on the square of the distance would be produced.

Accordingly, each measured distance is supposed to be affected by a term d depending on the square of the distance, and by a term

$$= \Delta S \times \frac{\text{measured distance}}{\text{standard distance}}.$$

The tabular Right Ascensions of the preliminary star-places are supposed to be affected by errors that can be expressed by

$$ma + m^2b,$$

where m = the difference in magnitude of the star from 4·5 mag. (so that for 6th mag. $m = +1 \cdot 5$). The proper factors of these terms are formed for each comparison of the tabular distances (computed from the *preliminary places*) with the heliometer measures of the same distance.

In this way 70 equations are formed, involving the four unknown quantities

$$a, b, d, \text{ and } \Delta S.$$

Approximate assumptions are made as to the mean error of the preliminary Catalogue and that of the heliometer measure—the latter being derived from measures of 19 pairs of stars which have been observed more than once. The *mean error* of a heliometer measure was ±0″·22 corresponding with a probable error of ±0″·15.

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A system of weights was adopted such that the mean error of an equation of weight unity should be $\pm 0''\cdot 50$, and the solution showed a mean error of $\pm 0''\cdot 55$, so that the weights originally adopted were considered sufficiently accurate.

The average weight of an equation was 2·3. The solution of these 70 equations by least squares, having regard to the weight of each equation, leads to four normal equations, and a discussion of these shows that the terms b and d have no reality, since their omission from the equations makes no sensible increase of the sum of the squares of the residuals; but the omission of the term a increases the sum of the squares of the residuals from 23·0 to 36·8. When allowance is made for the errors of the heliometer measures, it is shown that the square of the mean discordance between the heliometer and meridian observations is reduced as $2\frac{1}{2}$ to 1 by the introduction of the correction depending on magnitude. The value of the term so determined is

$$a = -0''\cdot 27 \pm 0''\cdot 07,$$

or, in other words, *the Right Ascensions of the preliminary Catalogue have to be diminished fully quarter of a second of arc per magnitude fainter than* 4·5.

Applying this correction to the preliminary Right Ascensions, the R.A.'s of each Observatory are separately compared with the corrected Catalogue so formed, each comparison giving an equation of the form

$$a + mb + m^2 d = \mathrm{C} - \mathrm{O}.$$

Normal equations are then formed, and from these the values of a, b, and d are found for each Observatory.

Applying the corrections thus deduced, the whole of the observations are rediscussed and a definitive Catalogue is formed, corresponding systematically with the heliometer observations.

[The remarkable result comes out, viz. that the eye and ear observations correspond, within the limits of probable error, with the results of the heliometer measures; but the chronographic observations require large corrections depending on magnitude.]

With the star-places so found, and with Le Verrier's Tables of *Mars*, corrected for an assumed constant error of $+2''\cdot 45$ in heliocentric longitudes (derived from the Greenwich Meridian Observations), and employing $8''\cdot 80$ for the Mean Hor. Solar Parallax and $\frac{1}{300}$ for compression of the Earth, the apparent tabular distances of *Mars* from the comparison stars are computed and compared with the heliometer measures. From this comparison equations are formed on the type

$$f'\Delta\alpha + f''\Delta\delta + f'''n + \nu z = \mathrm{O} - \mathrm{C} + rm - f'(\tau - \tau_0)\kappa + f''(\tau - \tau_0)\kappa';$$

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where

f' and f'' are the factors in R.A. and Decl.
n, the unknown percentage of the error of the assumed parallax.
f''', the effect on the distance of increasing the parallax one per cent.
ν, $\frac{\text{The observed distance in seconds of arc}}{10000}$.
τ, the Greenwich mean time of observation expressed in days.
τ_0, the mean epoch of the combination to which each observation in that combination is reduced.
κ and κ', the daily rate of increase of the real over the tabular R.A. and Decl. at the epoch of observation.

These equations are then formed into combinations as follows:—

1. Each combination must consist of two or more *bunches* of observations. Those *bunches* must be symmetrical evening and morning, and must afford data for eliminating at least the scale value and parallax from their combination. *Bunches* are only to be considered corresponding and symmetrical when the comparison stars have been the same and have been observed in the same way in the evening and morning.

2. The mean epoch of each *bunch* must be within twenty-four hours of the mean epoch (τ_0) of the combination, since κ and κ' may legitimately be considered uniform during that period.

3. The observations within each *bunch* must be made without much interruption by cloud, &c., so that the scale value may be considered uniform throughout each *bunch*.

Subject to these conditions it was found that twenty-five combinations could be formed from the observations, but of these three, from unfavourable conditions, could not be included in the definitive result, though their introduction produces no sensible change in the resulting parallax or its probable error.

The following are the steps of reduction:—

1. Normal equations are formed for each *bunch*.

2. z is eliminated from the normals for each bunch, preserving the symmetry of the equations.

3. The normals of the same name in each combination are added together and so normals in $\Delta\alpha$, $\Delta\delta$, and n are found for each combination. From these $\Delta\alpha$ and $\Delta\delta$ are then eliminated, and an equation in n, with the coefficient of the true weight of n, is obtained for each combination, in terms of κ and κ'.

4. Neglecting κ and κ' an approximate value of n ($= -0{\cdot}25$) is obtained, and when this is substituted in the normals $\Delta\alpha$ and $\Delta\delta$, the values of $\Delta\alpha$ and $\Delta\delta$ are obtained for the epoch of each combination.

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5. It is then found that the separate determinations of Declination may be represented by

$$\Delta\delta = -0''.66 + .0059\,t + 0.00077\,t^2,$$

(where t is the time, in days, from September $5^d.5$) with a probable error of only

$$\pm 0''.057$$

for each determination of declination.

But the Right Ascensions cannot be represented so accurately. There is a strangely marked tendency in the run of the values of $\Delta\alpha$ indicating a variation having a period of about 13·6 days. Some of the observations cannot be represented within reasonable limits without the supposition of such a curve, but the general run of the observations is expressed by

$$\Delta\alpha = +0''.44 - .0114\,t - .00112\,t^2 \qquad (1)$$

But *all* the observations are well represented by the expression

$$\left.\begin{array}{lll} \Delta\alpha = & +0''.43 & \pm 0''.07 \\ & + .0035\,t & \pm .0032 \\ & - .000367\,t^2 & \pm .00016 \\ & + \begin{pmatrix} 0.376 \\ \pm .071 \end{pmatrix} \times \sin(u + 122^\circ.3 \pm 12^\circ.6) & \\ \text{where } u = & \dfrac{t}{13^d.626 \pm .067} \times 360 & \end{array}\right\} (2)$$

and the probable error of each determination of $\Delta\alpha$ is $\pm 0''.199$.

[The coincidence between this period ($13^d.63$) and that of a term depending on twice the Moon's mean longitude is remarkable, but the author can find no grounds for such an inequality, nor does he consider the real existence of such a period to be as yet proved.]

6. The values of κ and κ' are then found for the epoch of each combination by the above expressions for $\Delta\alpha$ and $\Delta\delta$.

When κ is found by (1)

$$n = -.209, \quad \text{or } \pi = 8''.782 \pm 0''.0116.$$

When κ is found by (2)

$$n = -.251, \quad \text{or } \pi = 8''.778 \pm 0''.0119.$$

The probable errors of a single measure of distance are by (1)

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and (2) respectively $\pm''\cdot234$ and $\pm''\cdot241$, or about double the probable error of measures of stars and minor planets.

The author then examines the various sources of systematic error.

It is shown, by terms carried throughout the discussion, that if *no corrections for refraction* whatever had been employed, the resulting parallax would have been increased by $+0''\cdot014$, and as Bessel's refraction tables cannot be supposed $\frac{1}{100}$ part in error, no conceivable error in the refractions can produce an error of $0''\cdot00014$ in the deduced parallax. This is due to the symmetrical arrrngement of the comparison stars, and it is shown by an independent discussion, that even when the error of scale value is not eliminated from every bunch, as in the definitive reduction, such is the general symmetry of arrangement of the comparison stars that the value of the parallax comes out the same, and with practically the same probable error.

Mr. Gill comes to the conclusion, after an examination of every source of systematic error, that the only systematic error to which the result can be supposed liable, may be a small error due to chromatic dispersion of the atmosphere, but he also shows that this is probably very small from the close agreement between the resulting parallaxes at great and lesser zenith distances.

The following are the separate results:—

When only symmetrical combinations are employed (definitive discussion):

From symmetrical observations combined ...	$8''\cdot777 \pm 0''\cdot013$
Rejecting 3 combinations observed under unsatisfactory circumstances... ...	$8\cdot782 \pm 0\cdot012$
Rejecting 3 combinations on the supposition of a periodic variation of the R.A.	$8\cdot778 \pm 0\cdot012$

When the scale value is supposed constant, its error not being eliminated as in the previous discussion:

From all symmetrical observations combined ...	$8''\cdot778 \pm 0''\cdot014$
Rejecting 3 doubtful combinations ...	$8\cdot784 \pm 0\cdot013$

When the whole of the observations are combined, and the star-places are regarded as absolutely known:

From all the observations combined	$8''\cdot783 \pm 0''\cdot026$
Observations of greater zenith distance.	$8\cdot786 \pm \cdot034$
„ lesser zenith distance...	$8\cdot780 \pm \cdot040$
„ before opposition	$8\cdot791 \pm \cdot034$
„ after opposition ...	$8\cdot772 \pm \cdot040$

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The separate results of the definitive discussion are:—

Date.		π	Dif. from Mean.	Weight of n.
1877, July	31 E and M	8″·90	+0″·12	·113
Aug.	5 E and M	·65	− ·13	·086
	12 E, Aug. 10 M	·80	+ ·02	·175
	14 E, Aug. 13 and 14 M	·83	+ ·05	·180
	15 E and M	·80	+ ·02	·079
	16 E and M	·90	+ ·12	·091
	19 E and M	·88	+ ·10	·109
	24 E and M	·77	− ·01	·227
	29 E and M	·84	+ ·06	·113
Sept.	4, 5, 6, E and M	·82	+ ·04	·547
	6 E and M	·67	− ·11	·057
	6 E and M	·62	− ·16	·097
	7 E and M	·83	+ ·05	·060
	8 E and M	·81	+ ·03	·134
	8 E and M	·70	− ·08	·097
	9 E and M	·83	+ ·05	·145
	10 E, Sept. 9 E and M	·65	− ·13	·309
	10 E 10 M	·66	− ·12	·106
	19 E and M	·76	− ·02	·077
	24 E and M	·71	− ·07	·109
	27 E Sept. 25 M	·86	+ ·08	·147
Oct.	3 E and M	·81	+ ·03	·110

The definitive result of the whole discussion gives

$$\pi = 8''\cdot78 \pm 0''\cdot012,$$

which corresponds with 93,080,000 miles for the mean distance of the Earth from the Sun.

This value of π is compared with that from the recent accurate determinations of the velocity of light by Cornu and Michelson; when this velocity is combined, on the one hand, with the constant of aberration, and, on the other, with the time occupied by light in traversing a mean radius of the Earth's orbit as derived from observations of *Jupiter's* satellites.

Michelson's determination of the velocity of light combined with Struve's constant of aberration gives

$$\pi = 8''\cdot81.$$

Cornu's determination combined with the same constant gives

$$\pi = 8''\cdot80.$$

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But the mean of all the best modern determinations of the constant of aberration gives, instead of Struve's value (20″·445), the somewhat larger value 20″·496, and Nyrén's discussion (*Mem. St. Petersburg Acad.* vol. xix.) of the best modern Pulkowa determinations (including Struve's) gives 20″·481.

Maclear's very accurate observations of the Declination of α and β *Centauri* at the Cape give 20″·53 and 20″·59 respectively for the constant of aberration, showing, so far as their evidence goes, that the constant of aberration is not less when determined from Southern than from Northern stars.

If, as other results would appear to show, Struve's constant of aberration should be increased by 0″·05, then the resulting solar parallax from the combination with this constant would be diminished by 0″·02, and the values of π would be

According to Cornu		8″·78,
	Michelson	8·79,

values practically identical with the results derived from the heliometer observations of *Mars*.

Glasenapp's determination of the light equation (k) derived by him from eclipses of *Jupiter's* satellites gives

$$k = 500^{s}\!\cdot\!8 \pm 1^{s}\!\cdot\!02,$$

and this combined with Michelson's and Cornu's velocity of light gives

$$\pi = 8''\!\cdot\!76.$$

But if Glasenapp's determination of k is diminished in the direction of Delambre's value by the amount of its probable error—viz. 1ˢ—then the resulting parallax becomes

$$\pi = 8''\!\cdot\!78.$$

The large values of π which have been determined from meridian observations of *Mars* are explained by the fact that, at the altitudes at which the planet was observed in Europe in 1877, the limbs of *Mars* are spread by chromatic dispersion of the atmosphere into a fringe of coloured light 2″·2 broad. For the upper limb the violet part of the fringe is projected outside the disk of the planet, and for the lower limb the red part of the fringe is outside. In the field of a meridian instrument, illuminated by white light, more of the violet than of the red false limb would be obliterated, and hence the observer will cut more deeply with the spider-web into the false violet than into the more glaring red limb. Thus all the zenith distances measured would be too great, and the resulting parallax would also be too great. The results of other methods of determining the parallax

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